Notes on Signal to Noise Ratio

M.Lampton
FUEGO Team
Space Sciences Lab
UC Berkeley
May 2012; Jan and April 2014

Quantum Efficiency and Responsivity

- **Quantum Efficiency QE**: $0<QE<1$ is the probability that an incoming photon produces an electron output from a pixel
- Responsivity is that same concept, except put into amperes per watt rather than electrons per photon:
- **Responsivity R** $= (q_{electron} / hc) \cdot QE \cdot \lambda_{microns}$ where $0<QE<1$

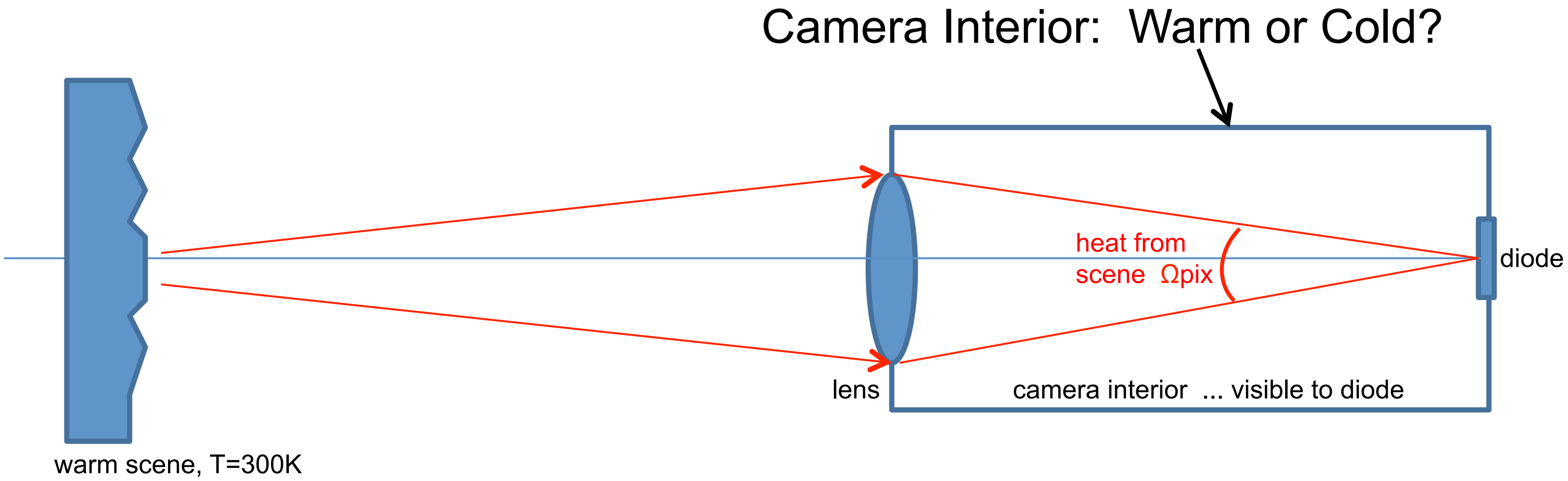
  $= 0.807 \cdot QE \cdot \lambda_{microns}$ amperes per watt
- Responsivity is used when converting an incident pixel scene power (watts) into sensor current, or accumulated charge in exposure time T_{exp}
- Signal output current $I_{sig} = R \cdot P_{sig} = \text{Responsivity}_{A/W} \cdot P_{sig}$
- Signal output charge $Q_{sig} = R \cdot P_{sig} \cdot T_{exp} = \text{Responsivity}_{A/W} \cdot E_{sig}$

Signal to Noise Ratio and NEP from Current Noise

- Suppose a sensor system is perfectly steady: no pointing jitter
- Suppose its scene is steady plus some small signal to be detected
- The output current of a pixel will be $I = I_{dark} + R \cdot (P_{scene} + P_{signal}) \pm I_{rms}$
  - R = Responsivity of pixel, in amperes per watt
  - $P = P_{scene} + P_{signal}$ = radiant power within Δλ arriving at the pixel, in watts
  - I_{rms} is the root mean square noise in the pixel current measurement
- Here, I_{DC} = dark + scene currents are assumed perfectly subtractible
  - from many statistically independent estimates over time and space
- For one sample, I_{rms} = pixel current noise
  - This is the one-sigma current measurement uncertainty for one pixel, one sample
- Noise Equivalent Power: **NEP** ≡ Prms = I_{rms} / Responsivity
- Signal to Noise Ratio: **SNR** = P_{signal}/NEP = Responsivity $\cdot P_{signal}/I_{rms}$

Two Models for Photodetector Heat Environment

assuming now that detector is cold enough that Idark is negligible

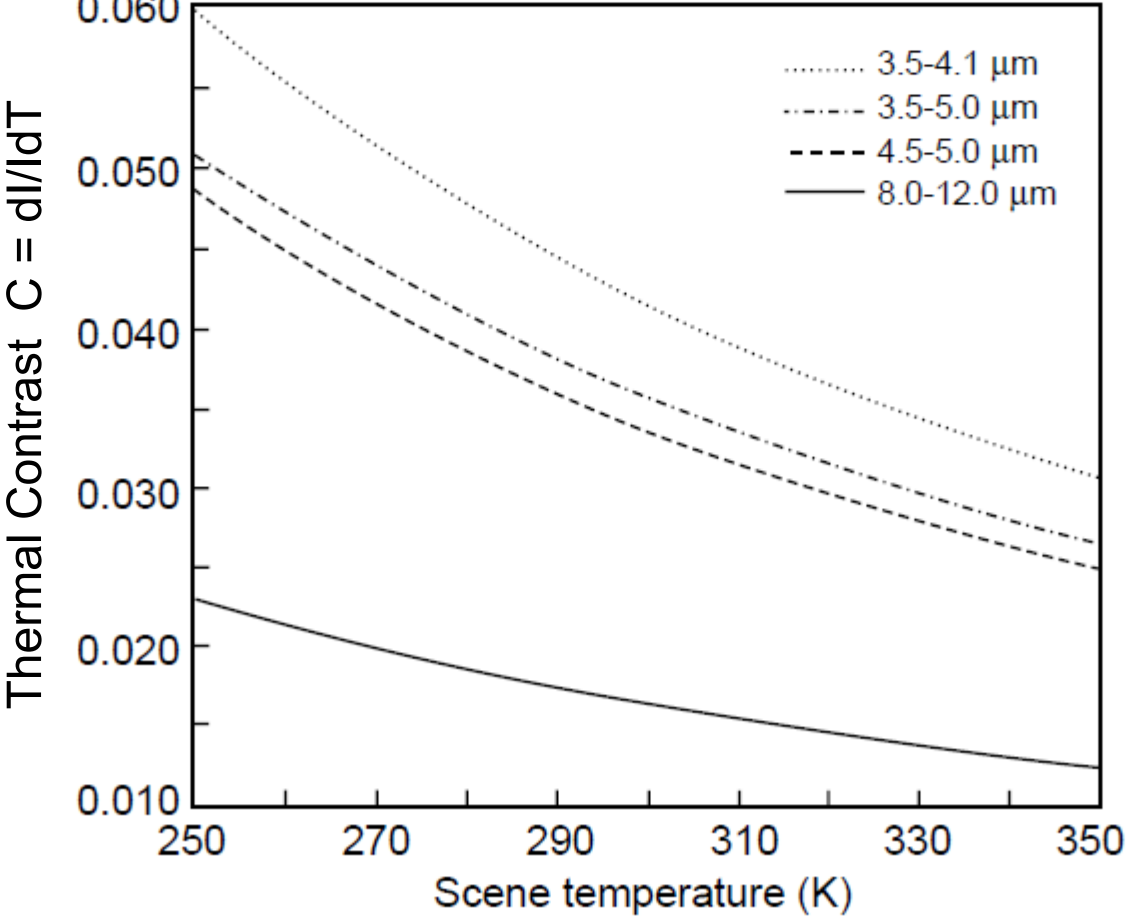


- Uniform scene, T=300K, **spatially** fills pixel diode area Apix
- Uniform scene heat fills optical **solid angle** Ωpix, typically 0.01 to 1 ster
- **Cold** camera interior (e.g. IR astronomy): no other added heat or noise.
- or **Warm** camera interior: 2π hemisphere, 300K, Ωeff=π ster.

Thermal contrast and NEDT for Photodiodes

- Given noise Irms, **NEP** = Irms/Responsivity = Irms/(dI/dP)
- Use slope dI/dT of Planck law: **NEDT** ≡ Trms = Irms/(dI/dT)

Thermal contrast is the relative change in thermal flux for a one kelvin change in temperature.



Rogalski, A., Progress in Quantum Electronics v.27 (2003) Fig 23

Shot Noise I_{rms} from I_{DC}

- There are many causes of noise: 1/f, microphonics, shot noise
- Shot noise dominates in high-brightness situations
- Shot noise is the statistically independent random arrivals of electrons
  - Steady dark current, if any
  - Steady current from camera interior heat, if any
  - Steady scene current
- The Poisson distribution describes the accumulation of random arrivals
- The variance of the Poisson distribution equals its mean
- $N_{rms} = \sqrt{(N_{mean})} = \sqrt{(I_{DC} \cdot T_{exp} / q_{electron})}$
- $Q_{rms} = \sqrt{(I_{DC} \cdot T_{exp} \cdot q_{electron})}$
- $I_{rms} = Q_{rms}/T_{exp} = \sqrt{(I_{DC} \cdot q_{electron} / T_{exp})}$
- This is often recast in terms of bandwidth $B=1/(2T_{exp})$:
- $I_{rms} = \sqrt{(2 I_{DC} \cdot q_{electron} \cdot B)}$

Relationships for thermal photons, current, noise

Cold Camera: use Ω = Ωoptical; Warm Camera use Ω = π steradians.

Astronomers use cold cameras to make the second term negligibly small.

Both cases here: I ignore dark current, i.e. the pixels themselves are plenty cold.

Photon arrival rate: pixel area A, optical solid angle Ω, and waveband $\Delta\lambda$:

$$N = A \cdot \Omega \cdot \Delta\lambda \cdot \frac{2c}{\lambda^4 \cdot (exp(hc/\lambda kT) - 1)}$$

Total DC $= I_{scene} + I_{camera}$ given QE_{opt}, T_{scene}, QE_{diode} and T_{camera}:

$$I_{dc} = \frac{q \cdot QE_{opt} \cdot A \cdot \Omega \cdot \Delta\lambda_{opt} \cdot 2c}{\lambda^4 \cdot (exp(hc/\lambda kT_{scene}) - 1)} + \frac{q \cdot QE_{diode} \cdot A \cdot \pi \cdot \Delta\lambda_{diode} \cdot 2c}{\lambda^4 \cdot (exp(hc/\lambda kT_{camera}) - 1)}$$

RMS current noise for an integrating exposure time $\tau_{exp} = 1/(2B_{Hz})$:

$$I_{rms} = (2qI_{dc}B_{Hz})^{0.5} = (qI_{dc}/\tau_{exp})^{0.5}$$

Derivative of current with respect to scene temperature:

$$\frac{dI_{scene}}{dT_{scene}} = I_{scene} \cdot \frac{hc \cdot exp(hc/\lambda kT_{scene})}{\lambda kT_{scene}^2 \cdot (exp(hc/\lambda kT_{scene}) - 1)}$$

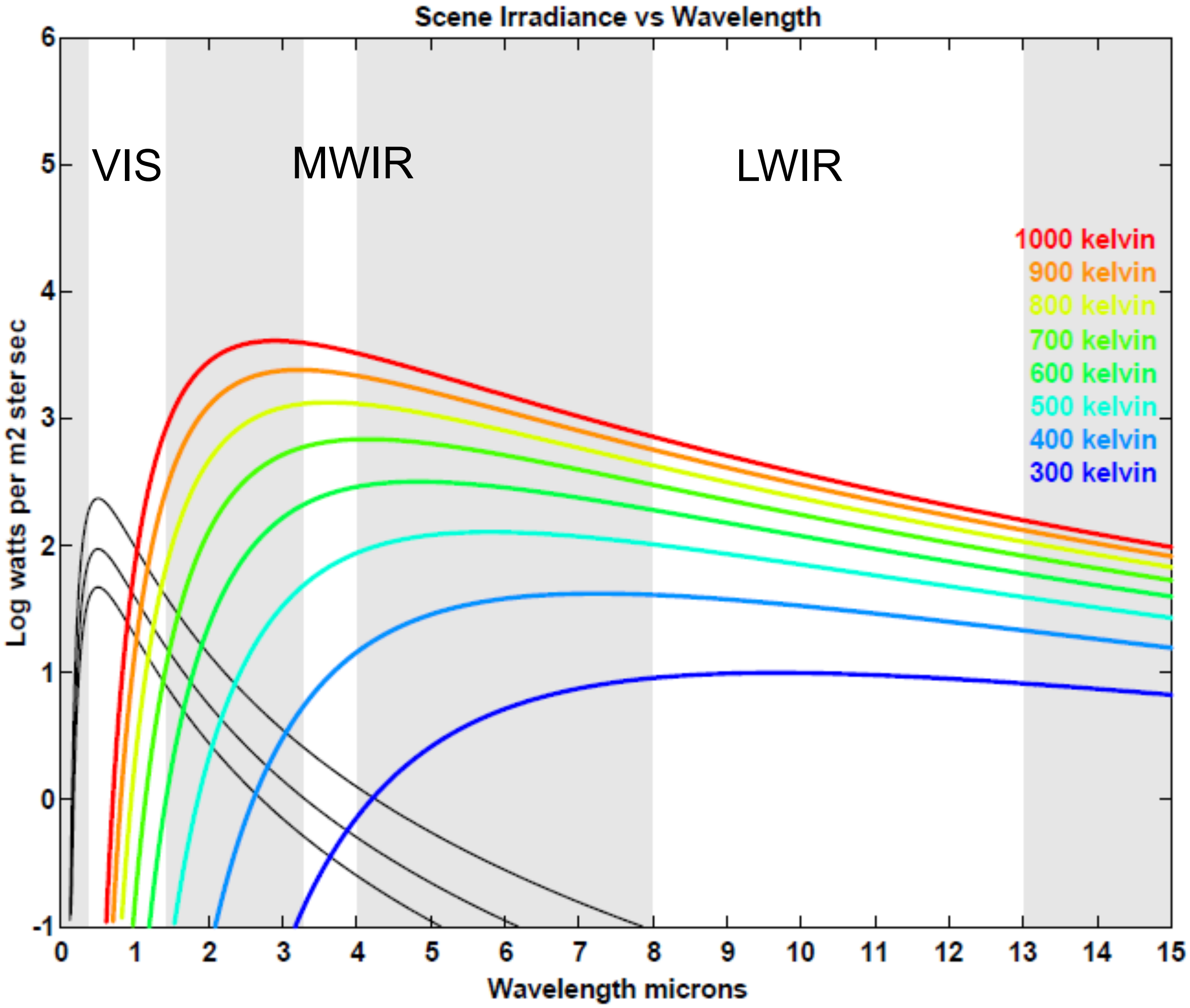
Specific Detectivity D*

- NEP_{watts} is the incoming power at a pixel required to yield SNR=1
  - for a microbolometer, 15µm pixel, 100Hz: NEP ≈ 1E-10 watts.
- Detectivity D = 1/NEP = Responsivity/I_{rms}
- Usually, I_{rms} is proportional to square root of pixel area A
  - Dark current is linearly proportional to A, other things being equal
  - Scene current is linearly proportional to A, other things being equal
- Usually, I_{rms} is proportional to square root of electrical bandwidth B
- Specific Detectivity D* factors out these two proportionalities:
- $D^* = D \cdot \sqrt{A} \cdot \sqrt{B} = \sqrt{A} \cdot \sqrt{B} / NEP$
  - for a microbolometer, D* ≈ 1E8 cm rootHz/watt.
- In this way, D* describes the sensivity of a sensor normalized to a common bandwidth (1 Hz) and a common area (1 cm2)
- D* is then is a material property dependent on wavelength, temperature, and radiative scene environment, but not dependent on integration time, bandwidth, or pixel size.
- D* has units of cm √Hz / watt.

Using D* to get SNR for bolometers

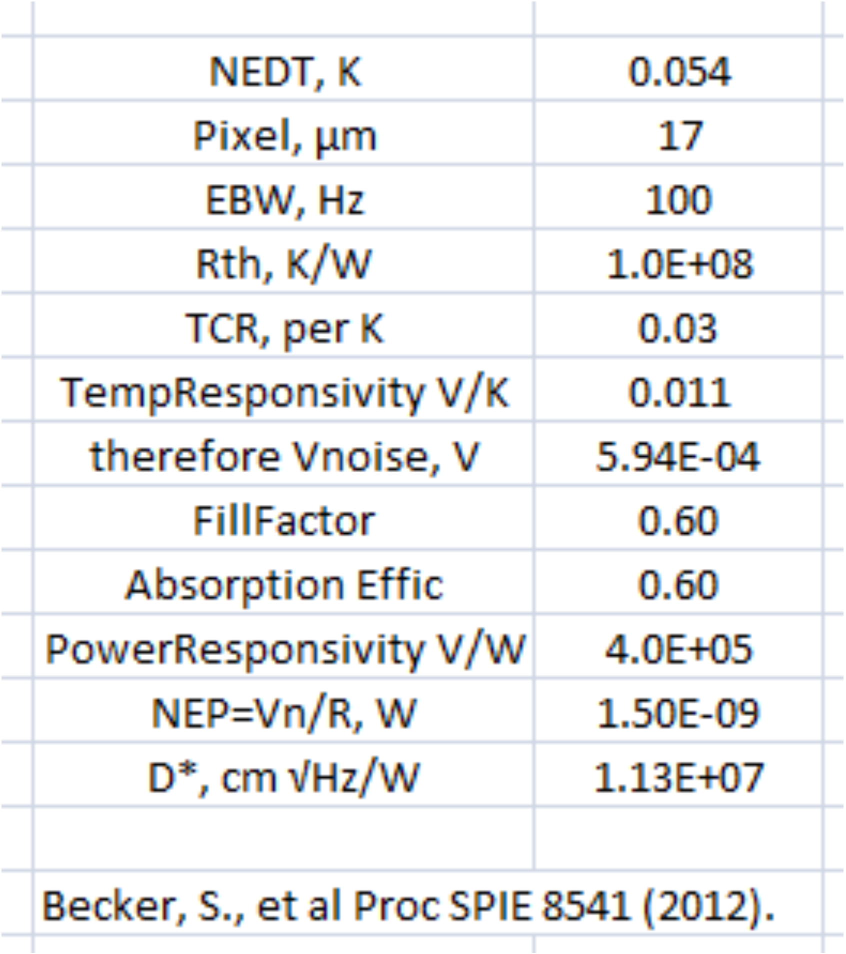
- Signal to noise ratio SNR = Psignal/Pnoise
- Pnoise = 1/D = √A · √B /D*
- So... SNR = Psignal D* /(√A · √B)
- If a microbolometer has pixel size 17µm, √A =0.00017 cm
- If a microbolometer has 100 frames/sec, √B = 7 √Hz
- If a microbolometer has D* = 1E8 cm rootHz/W, then
- SNR = Psignal/Pnoise with Pnoise=10^{-10} watts.

Scene Irradiance in Three Wavebands

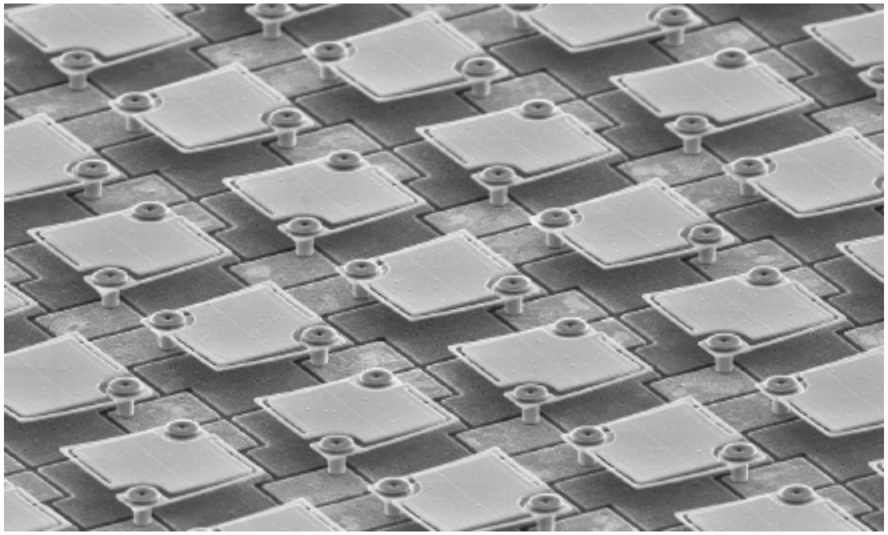


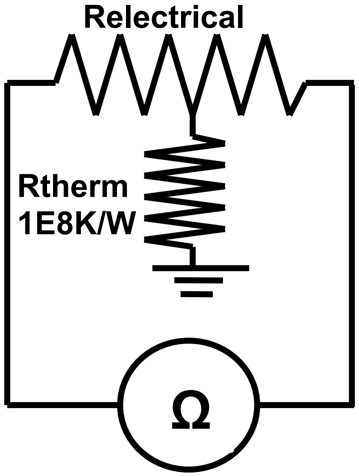
A Warm Microbolometer Model

based on the LETI/IR-FPA model



NEDT, K	0.054
Pixel, µm	17
EBW, Hz	100
Rth, K/W	1.0E+08
TCR, per K	0.03
TempResponsivity V/K	0.011
therefore Vnoise, V	5.94E-04
FillFactor	0.60
Absorption Effic	0.60
PowerResponsivity V/W	4.0E+05
NEP=Vn/R, W	1.50E-09
D*, cm √Hz/W	1.13E+07
Becker, S., et al Proc SPIE 8541 (2012).	



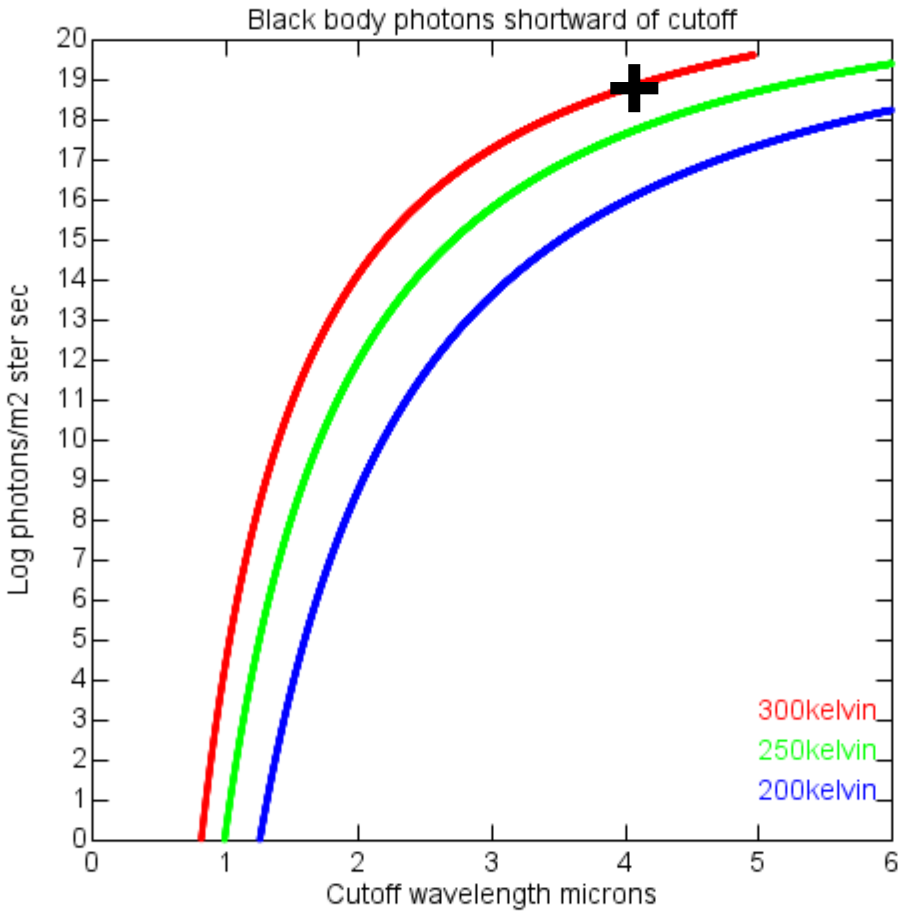


Elevated
thermistor
at 300K

Substrate
electronics
at 300K

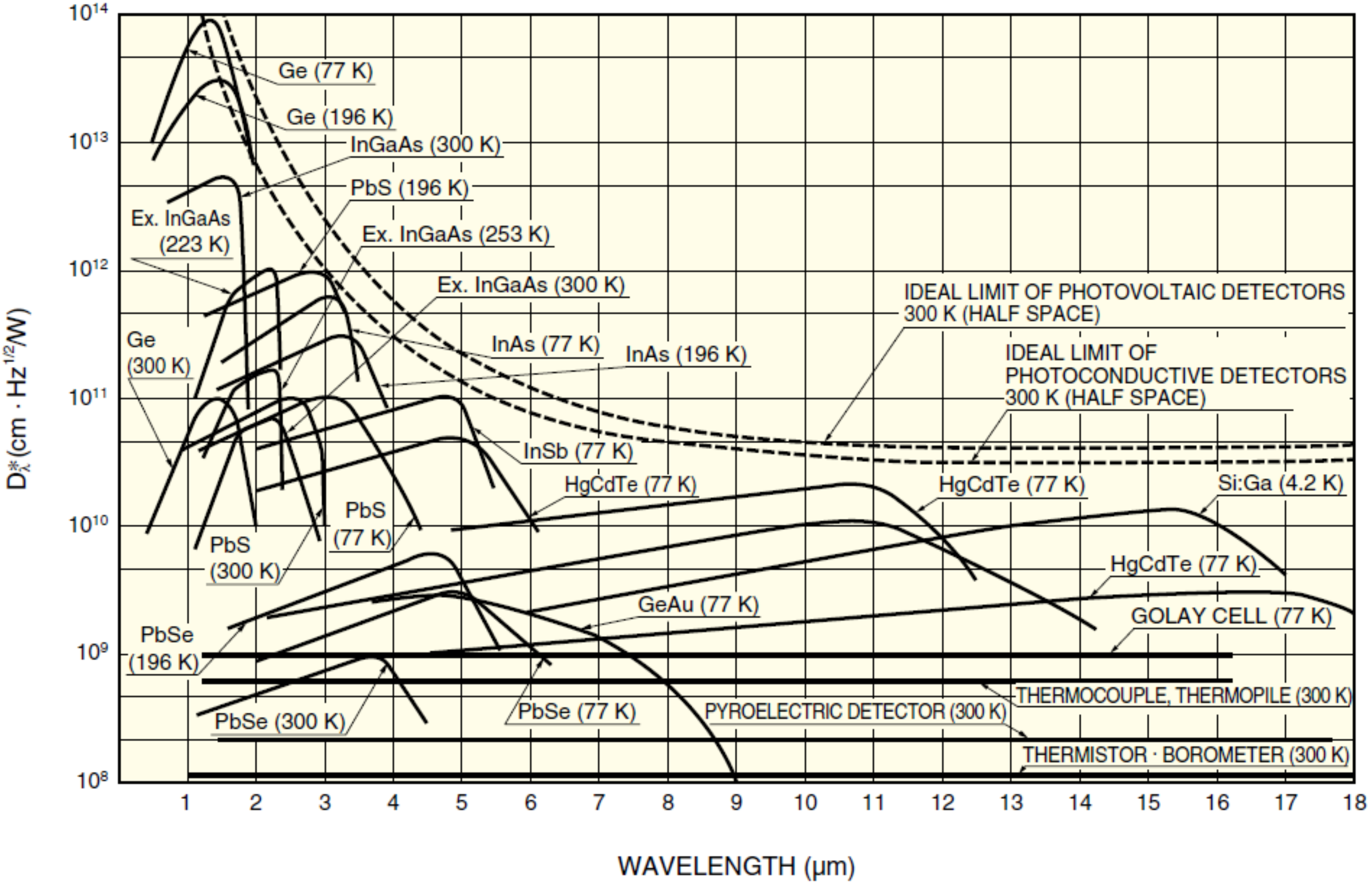
A Cold Photodiode Warm Camera Model

- Pixel size 15 µm; Ωview = π ster
- Dark current is entirely due to half space *heat* within camera
- This photon flux is the Planck law shortward of sensor cutoff : Nph=6E18 ph/m2.s.ster @ 4µm.
- DarkRate = Nph · Apix · Ωview = 4E9 events/sec.pixel
- RMS = √(Rate · Texp) photons
- NEP =signal at λ that gives a net photon count = RMS photons in Texp, i.e. NEP= (hc/ λ) · RMS/Texp
- NEP = (hc/ λ) · √(π Nph Apix /Texp)
- NEP = (hc/ λ) · √(2π Nph Apix B)
- D*=2.5E11 cm √Hz/W at 4µm.



D* Values for a wide variety of IR detectors: 300K Half Space

Characteristics and Use of Infrared Detectors," Technical Information Bulletin SD-12, Hamamatsu Inc. Fig 2.1

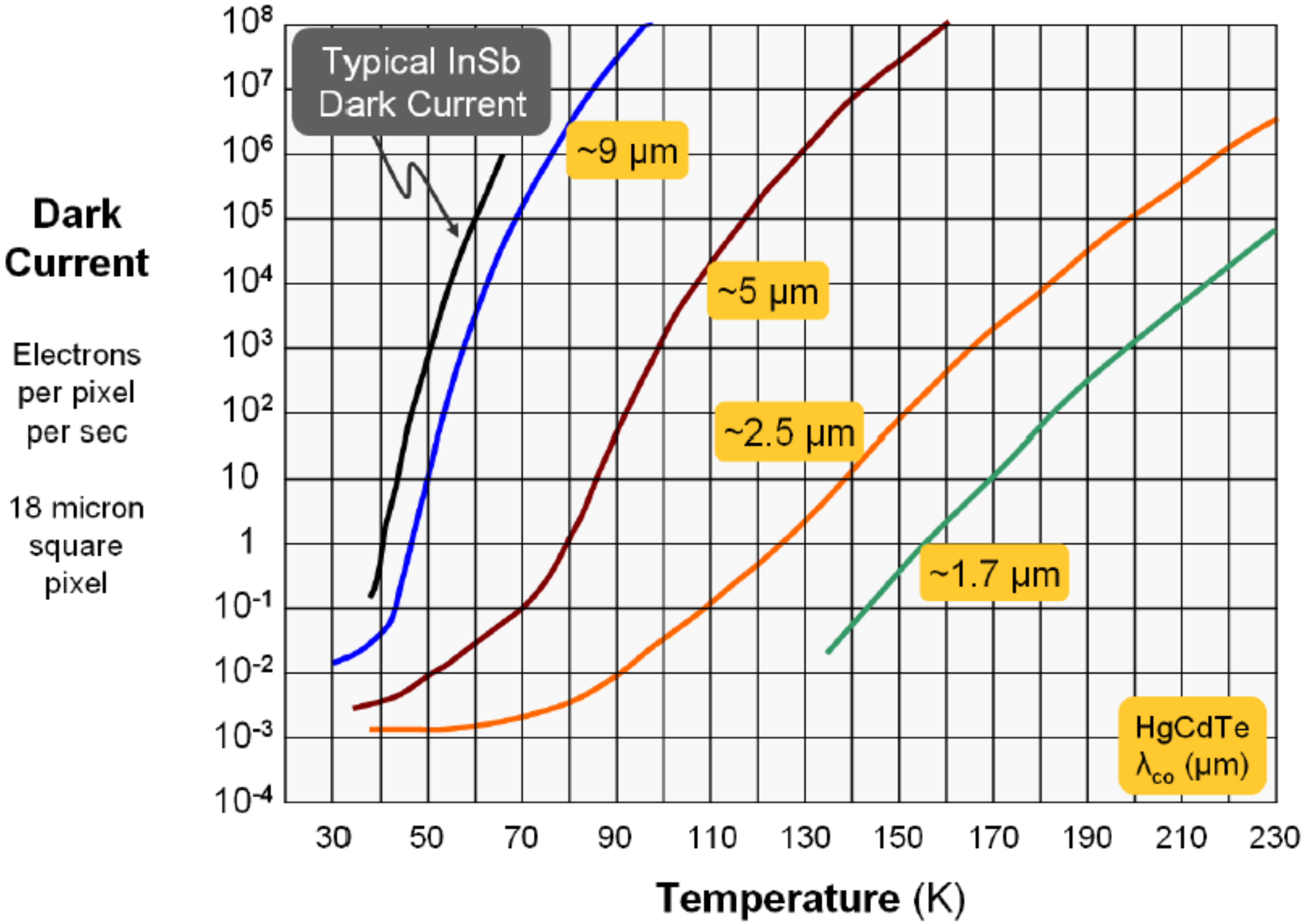


Of course for very sensitive work...

- Never flood your detector with heat!
- Always use a cold stop to block stray instrument heat!
- Keep the sensor environment as cold as possible!
- etc
- etc
- and after all that, it is best to model each heat source and combine the results to predict the system sensitivity.

InSb and MCT Imaging Detector Arrays

Beletic et al Proc SPIE 7021 2008



References to Current Work

Kozlowsky, L. & Kosonocky W., “Infrared Detector Arrays,” Chapter 33 of Handbook of Optics Vol 2, McGraw-Hill (2010).
Sizov, F., “IR region challenges: Photon or thermal detectors?” Semi.Phys. Quant. E&O, v15, 183-199 (2012).
Reibel, Y., et al., “Large format, small pixel pitch and hot detectors at SOFRADIR,” Proc SPIE v.8896 (2013).
Rogalski, A., “Infrared detectors: status and trends,” in Progress in Quantum Electronics v.27 (2003) .
Westerveldt, R., “Imaging Infrared Detectors II,” JASON Study JSR-97-600 (2000).
Baier, N., et al., “MCT planar p-on-n LW and VLW IRFPAs,” Proc SPIE v.8704 (2013).
Reibel, Y., et al., “High-performance and long-range cooled IR technologies in France,” Proc SPIE v.8704 (2013).
Reibel., Y., et al., “Small pixel pitch solutions for active and passive imaging,” Proc Spie v.8353 (2012).
Reibel, Y., et al., “Infrared dual-band detectors for next generation,” Proc SPIE v.8012 (2011).
Tribolet, P., et al., “Advanced HgCdTe technologies and dual-band developments,” Proc SPIE v.6940 (2008).