Notes on Signal to Noise Ratio

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Quantum Efficiency and Responsivity

• **Quantum Efficiency QE**: $0 < \text{QE} < 1$ is the probability that an incoming photon produces an electron output from a pixel.

• Responsivity is that same concept, except put into amperes per watt rather than electrons per photon:
  
  \[ \text{Responsivity } R = \left( \frac{q_{\text{electron}}}{hc} \right) \cdot \text{QE} \cdot \lambda_{\text{microns}} \quad \text{where } 0 < \text{QE} < 1 \]
  
  \[ = 0.807 \cdot \text{QE} \cdot \lambda_{\text{microns}} \text{ amperes per watt} \]

• Responsivity is used when converting an incident pixel scene power (watts) into sensor current, or accumulated charge in exposure time $T_{\text{exp}}$.

• Signal output current $I_{\text{sig}} = R \cdot P_{\text{sig}} = \text{Responsivity}_{A/W} \cdot P_{\text{sig}}$

• Signal output charge $Q_{\text{sig}} = R \cdot P_{\text{sig}} \cdot T_{\text{exp}} = \text{Responsivity}_{A/W} \cdot E_{\text{sig}}$
Signal to Noise Ratio and NEP from Current Noise

• Suppose a sensor system is perfectly steady: no pointing jitter
• Suppose its scene is steady plus some small signal to be detected
• The output current of a pixel will be \( I = I_{\text{dark}} + R \cdot (P_{\text{scene}} + P_{\text{signal}}) \pm I_{\text{rms}} \)
  – \( R \) = Responsivity of pixel, in amperes per watt
  – \( P = P_{\text{scene}} + P_{\text{signal}} \) = radiant power within \( \Delta \lambda \) arriving at the pixel, in watts
  – \( I_{\text{rms}} \) is the root mean square noise in the pixel current measurement
• Here, \( I_{\text{DC}} = \text{dark} + \text{scene currents} \) are assumed perfectly subtractible
  – from many statistically independent estimates over time and space
• For one sample, \( I_{\text{rms}} \) = pixel current noise
  – This is the one-sigma current measurement uncertainty for one pixel, one sample
• Noise Equivalent Power: \( \text{NEP} \equiv P_{\text{rms}} = I_{\text{rms}} / \text{Responsivity} \)
• Signal to Noise Ratio: \( \text{SNR} = P_{\text{signal}} / \text{NEP} = \text{Responsivity} \cdot P_{\text{signal}} / I_{\text{rms}} \)
Two Models for Photodetector Heat Environment
assuming now that detector is cold enough that Idark is negligible

• Uniform scene, T=300K, **spatially** fills pixel diode area Apix
• Uniform scene heat fills optical **solid angle** Ωpix, typically 0.01 to 1 ster
• **Cold** camera interior (e.g. IR astronomy): no other added heat or noise.
• or **Warm** camera interior: 2π hemisphere, 300K, Ωeff=π ster.
Thermal contrast and NEDT for Photodiodes

- Given noise Irms, **NEP** = Irms/Responsivity = Irms/(dI/dP)
- Use slope dI/dT of Planck law: **NEDT** ≡ Trms = Irms/(dI/dT)

Thermal contrast is the relative change in thermal flux for a one kelvin change in temperature.

Rogalski, A., Progress in Quantum Electronics v.27 (2003) Fig 23
Shot Noise $I_{\text{rms}}$ from $I_{\text{DC}}$

- There are many causes of noise: 1/f, microphonics, shot noise
- Shot noise dominates in high-brightness situations
- Shot noise is the statistically independent random arrivals of electrons
  - Steady dark current, if any
  - Steady current from camera interior heat, if any
  - Steady scene current
- The Poisson distribution describes the accumulation of random arrivals
- The variance of the Poisson distribution equals its mean
- $N_{\text{rms}} = \sqrt{\langle N_{\text{mean}} \rangle} = \sqrt{\langle I_{\text{DC}} \cdot T_{\text{exp}} / q_{\text{electron}} \rangle}$
- $Q_{\text{rms}} = \sqrt{\langle I_{\text{DC}} \cdot T_{\text{exp}} \cdot q_{\text{electron}} \rangle}$
- $I_{\text{rms}} = Q_{\text{rms}} / T_{\text{exp}} = \sqrt{\langle I_{\text{DC}} \cdot q_{\text{electron}} / T_{\text{exp}} \rangle}$
- This is often recast in terms of bandwidth $B = 1/(2T_{\text{exp}})$:
- $I_{\text{rms}} = \sqrt{2 \cdot I_{\text{DC}} \cdot q_{\text{electron}} \cdot B}$
Relationships for thermal photons, current, noise

**Cold Camera:** use $\Omega = \Omega_{\text{optical}}$; **Warm Camera** use $\Omega = \pi$ steradians.

Astronomers use cold cameras to make the second term negligibly small.

Both cases here: I ignore dark current, i.e. the pixels themselves are plenty cold.

Photon arrival rate: pixel area $A$, optical solid angle $\Omega$, and waveband $\Delta \lambda$:

$$N = A \cdot \Omega \cdot \Delta \lambda \cdot \frac{2c}{\lambda^4 \cdot (\exp(hc/\lambda kT) - 1)}$$

Total DC = $I_{\text{scene}} + I_{\text{camera}}$ given $QE_{\text{opt}}$, $T_{\text{scene}}$, $QE_{\text{diode}}$ and $T_{\text{camera}}$:

$$I_{dc} = \frac{q \cdot QE_{\text{opt}} \cdot A \cdot \Omega \cdot \Delta \lambda_{\text{opt}} \cdot 2c}{\lambda^4 \cdot (\exp(hc/\lambda kT_{\text{scene}}) - 1)} + \frac{q \cdot QE_{\text{diode}} \cdot A \cdot \pi \cdot \Delta \lambda_{\text{diode}} \cdot 2c}{\lambda^4 \cdot (\exp(hc/\lambda kT_{\text{camera}}) - 1)}$$

RMS current noise for an integrating exposure time $\tau_{exp} = 1/(2B_{Hz})$:

$$I_{rms} = (2qI_{dc}B_{Hz})^{0.5} = (qI_{dc}/\tau_{exp})^{0.5}$$

Derivative of current with respect to scene temperature:

$$\frac{dI_{\text{scene}}}{dT_{\text{scene}}} = I_{\text{scene}} \cdot \frac{hc \cdot \exp(hc/\lambda kT_{\text{scene}})}{\lambda kT_{\text{scene}}^2 \cdot (\exp(hc/\lambda kT_{\text{scene}}) - 1)}$$
Specific Detectivity D*

- \( \text{NEP}_{\text{watts}} \) is the incoming power at a pixel required to yield \( \text{SNR}=1 \)
  - for a microbolometer, 15\( \mu \text{m} \) pixel, 100Hz: \( \text{NEP} \approx 1\times10^{-10} \) watts.
- Detectivity \( D = 1/\text{NEP} = \text{Responsivity}/I_{\text{rms}} \)
- Usually, \( I_{\text{rms}} \) is proportional to square root of pixel area \( A \)
  - Dark current is linearly proportional to \( A \), other things being equal
  - Scene current is linearly proportional to \( A \), other things being equal
- Usually, \( I_{\text{rms}} \) is proportional to square root of electrical bandwidth \( B \)
- Specific Detectivity \( D^* \) factors out these two proportionalities:
  - \( D^* = D \cdot \sqrt{A} \cdot \sqrt{B} = \sqrt{A} \cdot \sqrt{B} / \text{NEP} \)
    - for a microbolometer, \( D^* \approx 1\times10^8 \) cm \( \sqrt{\text{Hz}}/\text{watt} \).
- In this way, \( D^* \) describes the sensitivity of a sensor normalized to a common bandwidth (1 Hz) and a common area (1 cm\( ^2 \))
- \( D^* \) is then a material property dependent on wavelength, temperature, and radiative scene environment, but not dependent on integration time, bandwidth, or pixel size.
- \( D^* \) has units of cm \( \sqrt{\text{Hz}} / \text{watt} \).
Using D* to get SNR for bolometers

- Signal to noise ratio $\text{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}}$
- $P_{\text{noise}} = \frac{1}{D} = \sqrt{A} \cdot \sqrt{B} / D^*$
- So... $\text{SNR} = \frac{P_{\text{signal}} D^*}{(\sqrt{A} \cdot \sqrt{B})}$
- If a microbolometer has pixel size $17\mu m$, $\sqrt{A} = 0.00017$ cm
- If a microbolometer has 100 frames/sec, $\sqrt{B} = 7$ $\sqrt{Hz}$
- If a microbolometer has $D^* = 1E8$ cm $\sqrt{Hz}/W$, then
- $\text{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}}$ with $P_{\text{noise}}=10^{-10}$ watts.
Scene Irradiance in Three Wavebands
**A Warm Microbolometer Model**

*based on the LETI/IR-FPA model*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEDT, K</td>
<td>0.054</td>
</tr>
<tr>
<td>Pixel, μm</td>
<td>17</td>
</tr>
<tr>
<td>EBW, Hz</td>
<td>100</td>
</tr>
<tr>
<td>$R_{th}$, K/W</td>
<td>$1.0\times10^8$</td>
</tr>
<tr>
<td>TCR, per K</td>
<td>0.03</td>
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<tr>
<td>TempResponsivity V/K</td>
<td>0.011</td>
</tr>
<tr>
<td>therefore $V_{noise}$, V</td>
<td>$5.94\times10^{-4}$</td>
</tr>
<tr>
<td>FillFactor</td>
<td>0.60</td>
</tr>
<tr>
<td>Absorption Effic</td>
<td>0.60</td>
</tr>
<tr>
<td>PowerResponsivity V/W</td>
<td>$4.0\times10^5$</td>
</tr>
<tr>
<td>$NEP=V_n/R$, W</td>
<td>$1.50\times10^{-9}$</td>
</tr>
<tr>
<td>$D^*$, cm vHz/W</td>
<td>$1.13\times10^7$</td>
</tr>
</tbody>
</table>


![Diagram showing relectrical and elevated thermistor at 300K, with substrate electronics at 300K](image)

M.Lampton April 2014
A Cold Photodiode Warm Camera Model

- Pixel size 15 μm; Ωview = π ster
- Dark current is entirely due to half space *heat* within camera
- This photon flux is the Planck law shortward of sensor cutoff: Nph=6E18 ph/m2.s. ster @ 4μm.
- DarkRate = Nph · Apix · Ωview = 4E9 events/sec.pixel
- RMS = √(Rate · Texp) photons
- NEP = signal at λ that gives a net photon count = RMS photons in Texp, i.e. NEP = (hc/λ) · RMS/Texp
- NEP = (hc/λ) · √(π Nph Apix /Texp)
- NEP = (hc/λ) · √(2π Nph Apix B)
- D*=2.5E11 cm νHz/W at 4μm.

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D* Values for a wide variety of IR detectors: 300K Half Space

Characteristics and Use of Infrared Detectors,” Technical Information Bulletin SD-12, Hamamatsu Inc. Fig 2.1
Of course for very sensitive work...

- Never flood your detector with heat!
- Always use a cold stop to block stray instrument heat!
- Keep the sensor environment as cold as possible!
- etc
- etc
- and after all that, it is best to model each heat source and combine the results to predict the system sensitivity.
InSb and MCT Imaging Detector Arrays

Beletic et al Proc SPIE 7021 2008

**Dark Current**
- Electrons per pixel per sec
- 18 micron square pixel

**Typical InSb Dark Current**
- \( \sim 9 \mu m \)
- \( \sim 5 \mu m \)
- \( \sim 2.5 \mu m \)
- \( \sim 1.7 \mu m \)

**HgCdTe \( \lambda_{co} (\mu m) \)**

**Temperature (K)**

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References to Current Work